Gaussian Process Modelling of Nonlinear Aerodynamic Forces Acting on Bluff Bodies

Igor Kavrakov [1], Allan McRobie [2], Guido Morgenthal [3]

[1]*University of Cambridge, UK, ik380@cam.ac.uk*
[2]*University of Cambridge, UK, fam20@cam.ac.uk*
[3]*Bauhaus-Universität Weimar, Germany guido.morgenthal@uni-weimar.de*

SUMMARY
Wind-induced vibrations are typically the critical design criterion for slender line-like structures. This work presents a data-driven model of the aerodynamic forces acting on bluff bodies based on Gaussian Process (GP) as a reverse-engineering machine learning method. The training input is designed as random harmonic motion consisting of vertical and rotational displacements. Once trained, the model is employed to predict both nonlinear dynamic forces and predict structural response during post-flutter behaviour. Applications include two benchmark bridge decks based on Computational Fluid Dynamics (CFD) data.

Keywords: Gaussian Processes, Data-driven, Bridge Aerodynamics, Aeroelasticity, Flutter.

1. INTRODUCTION

The semi-analytical models of the aerodynamic forces acting on bluff bodies such as bridge decks are still standard design practice. These models are however limited in their predictive capabilities for large-amplitude oscillations and strong turbulent winds; therefore, they are not able to capture strong aerodynamic nonlinear effects in the forces, such as high-order harmonics or simulate Limit Cycle Oscillations (LCOs). To tackle this, data-driven models based on e.g. Artificial Neural Network (ANN) have recently gained considerable attention (cf., e.g. Wu and Kareem, 2011; Abbas et al., 2020). These reduced-order models are initially trained based on data from CFD or experiments and can then be used to predict the aerodynamic forces or response. The wide mathematical properties of the data-driven models make them capable of capturing nonlinear aerodynamic phenomena (e.g. nonlinearity or non-stationarity) for a fraction of the computational time compared to the Computational Fluid Dynamics (CFD) models.

Alternatively to ANN for data-driven modelling, Gaussian Processes (GPs) (cf. Rasmussen and Williams, 2006) have enjoyed success as a machine learning method due to their non-parametric nature and ability to inherently consider measurement uncertainty. This study presents recent advancements in data-driven modelling of aerodynamic forces using GPs (cf. Kavrakov et al. (2022)). The model is completely non-dimensional, taking the effective angle of attack as input and having fluctuating lift and moment coefficients as output. The modelling framework is employed for two bridge decks to predict the nonlinear self-excited forces, critical flutter speed and post-flutter behaviour, i.e. LCOs.

2. GAUSSIAN PROCESS NONLINEAR AERODYNAMIC MODEL

Consider the wind-structure interaction system shown in Fig.1. The aerodynamic force model is constructed as a finite impulse response with exogenous input and additive independent identically distributed noise. For example, the lift force coefficient C_{Li} at discrete time-step i is formulated as:

$$C_{Li} = f_L(\boldsymbol{\alpha}_i) + \epsilon_i = f_L\left(\alpha'_{h,i}, \alpha'_{a,i}, \alpha_{a,i}, \alpha_{h,i}, \alpha_{a,i}, \alpha_{h,i-1}, \alpha_{a,i-1}, \dots, \alpha_{h,i-S}, \alpha_{a,i-S}\right) + \epsilon_i, \qquad (1)$$

where the input vector $\boldsymbol{\alpha}_i$ is constituted from the angles based on the vertical $\alpha_h = \operatorname{atan}(h'/B)$ and rotational motion $\alpha_a = \alpha$. The number of lag terms is denoted as S and the derivative w.r.t. non-dimensional time $\tau = tU/B$ is denoted with a prime.

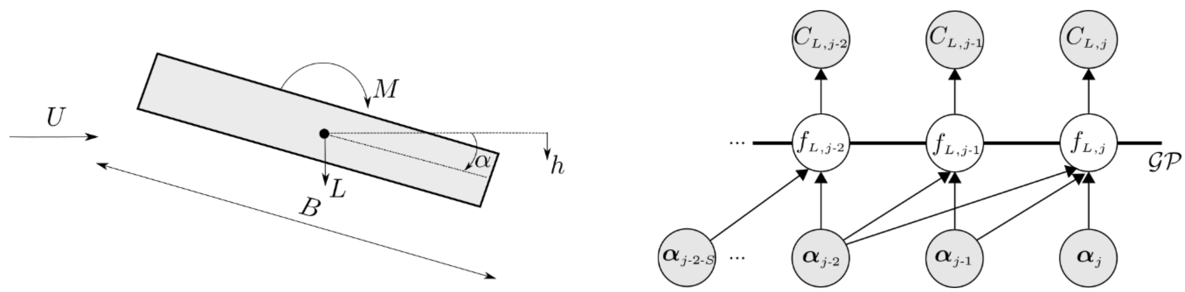

Figure 1. Wind-structure interaction system (left). Graphical representation of the GP model (right).

The nonlinear function f_L is a GP $f = \mathcal{GP}\left(m_f, k_L\right)$, with a covariance function $k = k(\boldsymbol{\alpha}_i, \boldsymbol{\alpha}_j; \boldsymbol{\theta})$ based on the input $\boldsymbol{\alpha}$ and hyperparameters $\boldsymbol{\theta}$ (which may include parameters from m_f), with Gaussian noise $\varepsilon = \mathcal{N}(0, \sigma_L^2)$. We use an exponential kernel with automatic relevance detection to construct the covariance matrix $\boldsymbol{K}_{LL}$ since it is infinitely differentiable, and particularly useful for dynamical problems (cf. Rasmussen and Williams, 2006).

Learning the nonlinear function, i.e. hyperparameters $\boldsymbol{\theta}$ is through minimization of the log-likelihood. Once these are learn, the predictive distribution at prediction points $\boldsymbol{\alpha}_i^*$ is

$$p(\ \mathbf{f}_{\mathrm{L}}^* | \boldsymbol{\alpha}^*, \boldsymbol{C}_L, \boldsymbol{\alpha}; \boldsymbol{\theta}) \sim N(\boldsymbol{K}_{LL^*}^T (\boldsymbol{K}_L + \sigma_L^2 \boldsymbol{I})^{-1} (\boldsymbol{C}_L - \boldsymbol{m}_f), \boldsymbol{K}_{L^*} - \boldsymbol{K}_{LL^*}^T (\boldsymbol{K}_L + \sigma_L^2 \boldsymbol{I})^{-1} \boldsymbol{K}_{LL^*}), \qquad (4)$$

where $\boldsymbol{K}_{LL^*}$ and $\boldsymbol{K}_{L^*}$ are constructed based on the prediction points. Similarly, the moment coefficient can be found.

The prediction framework is shown in the figure below:

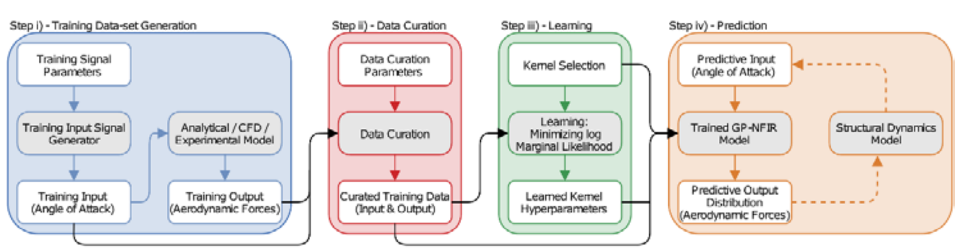

Figure 2. Prediction framework for the aerodynamic forces using a GP-NFIR model. Gray box - model; white box - input/output. The dashed line indicates if a dynamic model (aeroelastic analysis) is used for prediction.

3. APPLICATION

The methodology is applied to two bridge decks based on CFD data: the streamlined Great Belt Bridge deck and bluff H-shaped Tacoma-like section. The sections are shown below.

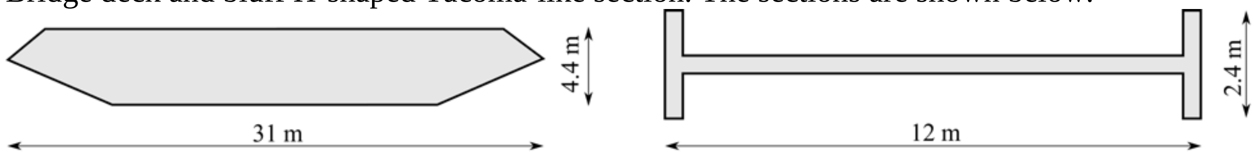

Figure 3. Streamlined Great Belt Deck (left); H-Shaped Tacoma-like Bluff Deck (right)

The input training motion consists of two separate signals stacked together with standard deviations of $\sigma_{\alpha_h}=\sigma_{\alpha_a}$=2.5 and 15 deg for the streamlined deck (Fig. 4, top), and $\sigma_{\alpha_h} = \sigma_{\alpha_a}$=1 and 8 deg for the bluff deck (Fig. 5, left). The reduced velocity range ($V_r = U/fB$; f=oscillation frequency) for the training signals is between $2 < V_r < 16$ and $2 < V_r < 8$ for the streamlined and bluff decks, respectively. The lift and moment GP models are trained separately, with the sample for the streamlined deck shown in Fig. 4 (bottom), and a sample for the moment coefficient for the bluff deck shown in Fig. 5 (left). The GP model fits the training signals well for the streamlined deck, but filters the violent vortex shedding from the CFD forces for the bluff deck.

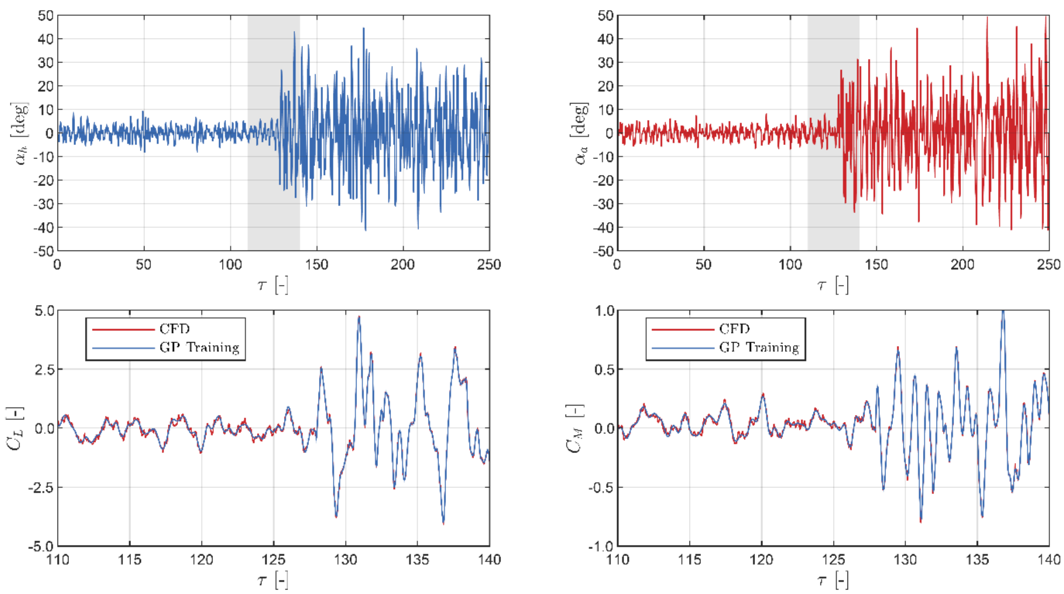

Figure 4. Streamlined Great Belt Deck: Training. Input angles (top); output sample moment and lift coefficients (top).

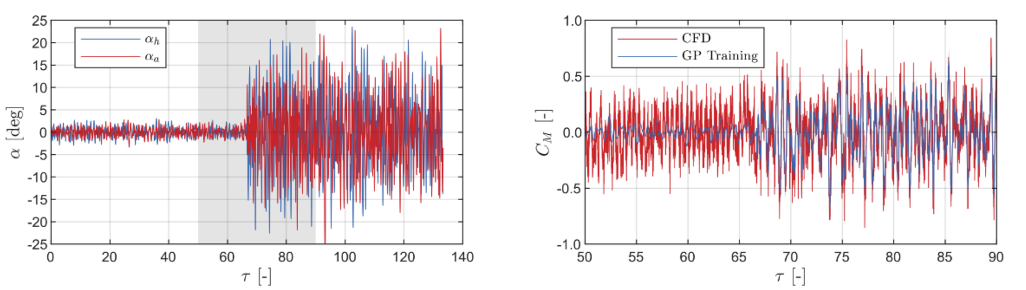

Figure 5. H-shaped Bluff Deck: Training. Input angles (left); output sample moment coefficient (right).

In the case of the streamlined deck, the trained GP model was used to predict the forces for large amplitudes (α_0 =15 deg) due to sinusoidal rotation. Fig. 6 (left) shows the moment coefficient and Fig. 6 (right) shows the corresponding Fourier spectrum. It can be seen that the GP model was able to accurately capture the forces, including the higher-order harmonics to a certain extent.

The GP model is used for the bluff deck to predict the LCO and flutter speed. Since the flutter is mostly torsional for this type of decks, Figure 7 depicts the rotation of both GP and CFD models. It can be observed that the GP model successfully predicted both the critical flutter velocity and LCO amplitude.

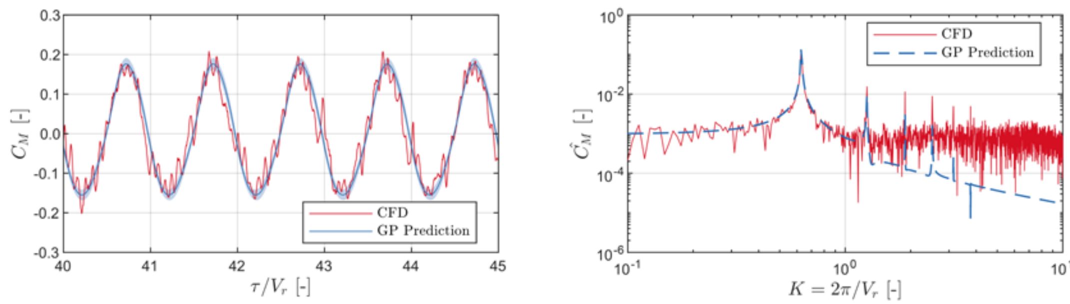

Figure **6**. Streamlined Great Belt Deck: Harmonic Force Prediction. Moment coefficient (left); Spectrum (right).

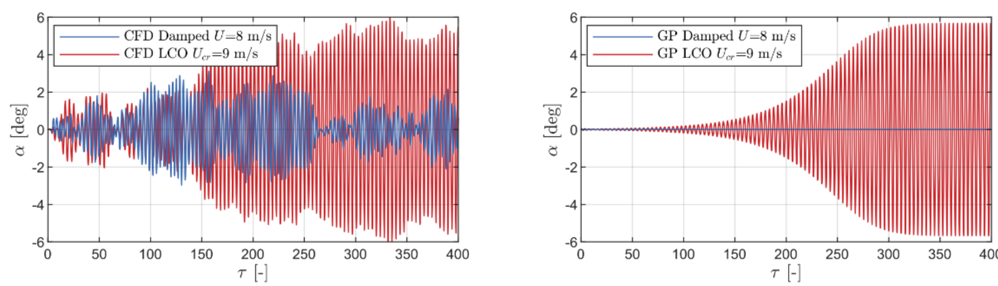

Figure **7.** H-shaped Bluff Deck: Flutter Prediction. Rotational DOF: CFD (left); GP (right).

4. CONCLUSION

We introduced a data-driven GP model for modeling aerodynamic forces on bluff bodies. The model was employed to predict the nonlinear forces for two bridge decks, including LCO amplitudes and higher-order harmonics. Potential applications of the presented framework can be in the structural analysis during the design and monitoring of linear structures.

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